

Comparing the Brooks and Gazzale Models

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June 25, 2002

Abstract

An attempt to find a common framework for our models.

1 Introduction

OK - not a lot of flashy intro here. I'll describe the Gazzale model, where producers locate in price/location space, then the Brooks model, where producers locate in category space, then tie the two together.

2 Gazzale model

The Gazzale model focuses on two producers competing for a heterogeneous consumer population. Producers choose price/location pairs, with a tension between locating near the 'sweet spot' and competing directly, and locating further out in the 'niches' and either acting as duopolists or as local monopolists.

I'll break the model into three pieces: consumer preferences, population structure, and producer behavior.

2.1 Consumer preferences

The product space can be thought of as a one-dimensional line, with coordinates on the line indicated with l 's.

Each consumer has a favorite location l - it will pay r (its reservation value) for goods at that location. As goods (or producer locations) get farther away from l , the consumer's value for that good falls off at a rate c . We assume that all consumers have the same r and c .

So each consumer can be described with the tuple $\langle l, r, c \rangle$.

2.2 Population structure

The consumer population is structured according to a tent distribution. There are α consumers at the 'sweet spot' l_0 , and this falls off at a rate γ as we move away from the sweet spot.

So we can describe the population structure with the tuple $\langle \alpha, \gamma \rangle$.

2.3 Producer Behavior

In each iteration, producers offer a single article at a single location. They each choose a location l and a price p for the article at that location. The only unknown is γ , which allows the producers to do a great deal of reasoning about where they should locate and price.

So each producer's offer can be described with the tuple $\langle l, p \rangle$.

3 Brooks model

In this section I'll describe the Brooks model in which producers search through category space. As in the previous section, I'll break it into consumer preferences, population structure and producer behavior.

3.1 Consumer preferences

We assume that there are a set c of categories which can be arranged on a line from c_1, \dots, c_j . Each consumer has a favorite category c^* , for which it is willing to pay w for each article in that category. Each consumer also values a fraction k of the categories; values fall off linearly from c^* .

Consumers also have a clutter cost $\alpha(|B|)$ that they pay for consuming a bundle B of size $|B|$. This is modeled as an exponential function $\alpha(|B|) = e^{\lambda|B|}$.

So we can characterize a consumer with the tuple $\langle w, k, c^*, \lambda \rangle$.

3.2 Population structure

The population is composed of a set N of niches. All consumers within a niche have the same w and k , c^* , and λ . There are no *a priori* restrictions on the number of niches, the size of each niche, or the relationship between niches.

We also allow the consumer population to change; every ξ iterations, a fraction ω of the consumers leave and are replaced with new, randomly generated consumers. Tournament selection is used to replace consumers; for each producer to remove, two are chosen randomly and the one with the lower surplus in the last ξ iterations is removed.

3.3 Producer behavior

On each iteration, each producer chooses a bundle, which consists of a number of articles in each category, along with a price for the bundle. If there are j categories, the producer's offer will consist of a vector of length $n + 1$: $\langle n_1, n_2, \dots, n_j, p \rangle$ indicating the number of articles in each category along with the price for the bundle. In addition, each category has a maximum size t ; this is the maximum number of articles in that category a producer will offer.

We assume very little producer knowledge about the structure of the consumer population. Producers select bundles, receive a profit signal, and act adaptively.

4 Relating the models

These models are actually quite similar; I think with a little pushing they can be mapped onto each other. Hopefully the previous layout has provided some hints as to how to do this.

4.1 Consumer preferences

This one is pretty easy; Gazzale consumers are described by the tuple $\langle l, r, c \rangle$, and Brooks consumers with the tuple $\langle w, k, c^*, \lambda \rangle$. Now, locations in the Gazzale model are the same thing as categories in the Brooks model, so c^* and l are the same thing. Similarly, willingness to pay is the same in both models, so w is the same as r . c in the Gazzale model is the rate at which valuation falls off; this is $\frac{w}{k}$ in the Brooks model. So, if we set $\lambda = 0$ in the Brooks model, these two formulations of consumer preferences are essentially the same.

4.2 Population generation

The Gazzale model describes a set of niches, where the niche at the sweet spot has α consumers and niches at a distance d from the sweet spot have $\alpha - (d*\gamma)$ consumers. This fits fine with the Brooks model, which just says that we have a bunch of consumers in different niches, but nothing specific about how those niches are related.

To make the models mesh, we'll leave out nonstationarity for the moment, setting $\omega = 0$ (or, equivalently, $\xi = \infty$).

4.3 Producer behavior

The Gazzale model has producers selecting $\langle l, p \rangle$ pairs, whereas the Brooks model has producers selecting $\langle n_1, n_2, \dots, n_j, p \rangle$ tuples. The obvious way to reconcile these is to limit the producer's bundle size to 1. Then they are choosing a single category (or location) in which they'll sell one article at price p to every willing consumer. This makes the models equivalent.

5 So What?

So we can explain these models in each other's terms; what does that give us? Well, it's nice to have a unifying framework to put everything under, even if it is *post hoc*.

More importantly, it should let us do some sanity checking. From the computational side, experimental results with the appropriate assumptions (one article, $\lambda = 0$, $\omega = 0$) should get the same results as the Gazzale model. From the analytic side, it suggests possible ways of extending the model, with the knowledge that we can easily generate experiments to guide this analysis. Some obvious extensions: what if a producer can offer 2 articles (or choose 2 locations) - does that encourage/discourage niche formation? What about clutter cost - it

seems that a little clutter cost helps with learning in the experimental results.
Can this be shown analytically?